Exercise 84

For the following exercises, for each polynomial, a. find the degree; b. find the zeros, if any; c. find the y-intercept(s), if any; d. use the leading coefficient to determine the graph's end behavior; and e. determine algebraically whether the polynomial is even, odd, or neither.

$$f(x) = -3x^2 + 6x$$

Solution

Part (a)

The degree of the polynomial is 2 because the highest power of x is 2.

Part (b)

Set $f(x) = 0$.

$$f(x) = -3x^2 + 6x = 0$$

Factor the polynomial.

$$3x(-x + 2) = 0$$

Use the zero product property.

$$3x = 0 \quad \text{or} \quad -x + 2 = 0$$

Solve each equation for x.

$$x = 0 \quad \text{or} \quad x = 2$$

Therefore, the zeros are

$$x = \{0, 2\}.$$

Part (c)

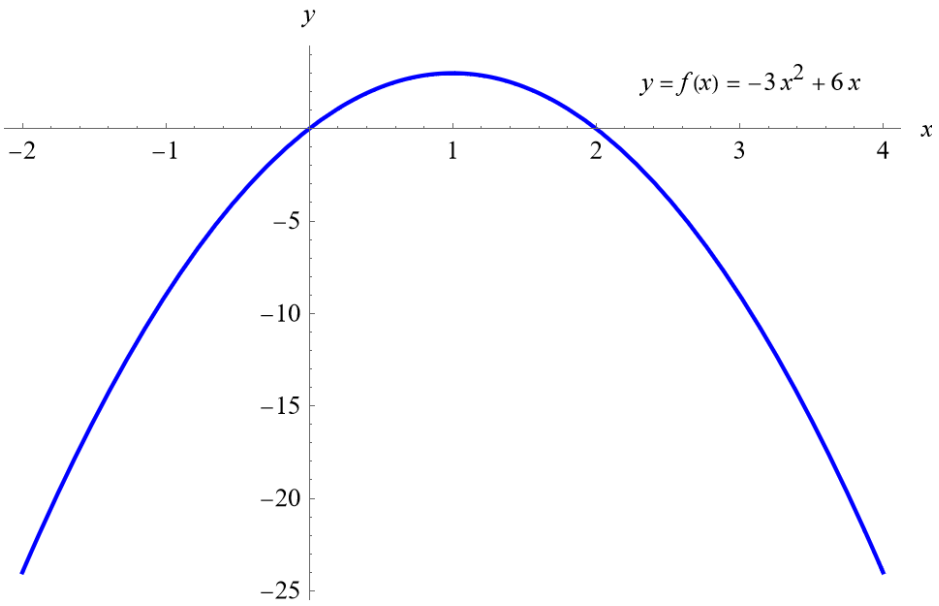
y-intercepts are the points where the function touches the y-axis, which occurs when $x = 0$.

$$f(0) = -3(0)^2 + 6(0) = 0$$

Therefore, there's one y-intercept: $(0, 0)$.

Part (d)

$-3x^2$ is the dominant term in the polynomial, so the graph is in the shape of a parabola. Since the coefficient is -3, it opens downward towards the negative y-axis. The graph of $f(x)$ versus x below illustrates this.



Part (e)

Plug in $-x$ for x in the function.

$$\begin{aligned} f(-x) &= -3(-x)^2 + 6(-x) \\ &= -3x^2 - 6x \end{aligned}$$

Since $f(-x) \neq f(x)$, the function $f(x)$ is not even.

Since $f(-x) \neq -f(x)$, the function $f(x)$ is not odd.